Sets and Counting

Finite Math

9 April 2019

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It is possible to have a set without any elements in it. We call this set the *empty set* or *null set*. We denote this set by $\varnothing$. An example of a set which is empty is the set of all people who have been to Mars.

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$$\{x|x \text{ is even}\}, \{x|x \text{ has hit more than 50 home runs in a single season}\}, \{z|z^2 = 1\}.$$

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The way to read this second type of set is, for example, “the set of x such that x is even” or “the set of z such that $z^2 = 1$. There are two kinds of sets: *finite sets* (the set only has finitely many elements) and *infinite sets* (the set has infinitely many elements). The sets $\{1, 2, 3, 4, 5...\}$ and $\{x|x \text{ is even}\}$ are infinite sets while the others are finite.

Example

Example

Let G be the set of all numbers whose square is 9.

(a) *Denote G by writing a set with a rule (the second style above).*

(b) *Denote G by listing the elements (the first style above).*

(c) *Indicate whether the following are true or false: $3 \in G$, $9 \in G$, $-3 \notin G$.*

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It follows that $\varnothing$ is a subset of every set and if $A \subset B$ and $B \subset A$, then $A = B$.

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Let $A = \{-3, -1, 1, 3\}$, $B = \{3, -3, 1, -1\}$, $C = \{-3, -2, -1, 0, 1, 2, 3\}$. *Decide the truth of the following statements*

$$\begin{array}{ccc} A = B & A \subset C & A \subset B \\ C \neq A & C \not\subset A & B \subset A \\ \varnothing \subset A & \varnothing \subset C & \varnothing \notin A \end{array}$$

Now You Try It!

Example

Let $A = \{0, 2, 4, 6\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, $C = \{2, 6, 0, 4\}$. *Decide the truth of the following statements*

$$\begin{array}{ccc} A \subset B & A \subset C & A = C \\ C \subset B & B \not\subset A & \varnothing \subset B \\ 0 \in C & A \notin B & B \subset C \end{array}$$

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$$\begin{array}{ccc} A \subset B & A \subset C & A = C \\ C \subset B & B \not\subset A & \varnothing \subset B \\ 0 \in C & A \notin B & B \subset C \end{array}$$

Solution

All true except for $B \subset C$.

Finding Subsets

Example

Find all subsets of the following sets:

(a) $\{a, b\}$

(b) $\{1, 2, 3\}$

(c) $\{\alpha, \beta, \gamma, \delta\}$

Set Operations

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To illustrate the set operations, we will use both actual sets and Venn diagrams. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4, 5\}$, and $B = \{3, 4, 5, 6, 7\}$.

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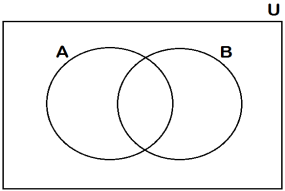

Unions

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Definition (Union)

The union of two sets A and B is the new set, denoted $A \cup B$, which consists of all elements which are in A or in B.

$$A \cup B = \{x | x \in A \text{ or } x \in B\}.$$

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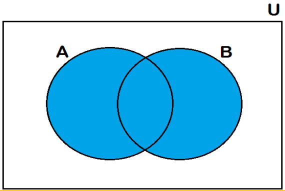

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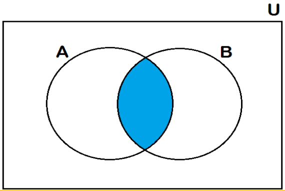

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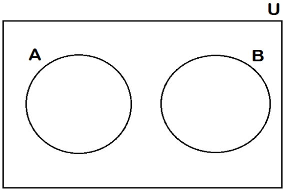

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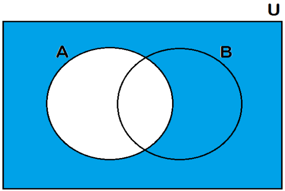

Examples

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Let $A = \{3, 6, 9\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{4, 5, 7\}$, *and* $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. *Find* $A \cup B$, $A \cap B$, $A \cap C$, *and* B'.

Now You Try It!

Example

Let $R = \{1, 2, 3, 4\}$, $S = \{1, 3, 5, 7\}$, $T = \{2, 4\}$, *and* $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. *Find* $R \cup S$, $R \cap S$, $S \cap T$, *and* S'.

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Solution

$$
\begin{aligned}
R \cup S &= \{1, 2, 3, 4, 5, 7\} \\
R \cap S &= \{1, 3\} \\
S \cap T &= \{2, 4\} \\
S' &= \{2, 4, 6, 8, 9\}
\end{aligned}
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$$\begin{aligned} n(A) &= 5 \\ n(B) &= 5 \\ n(A \cap B) &= 3 \\ n(A \cup B) &= 7 \\ n(A') &= \end{aligned}$$

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$$\begin{aligned} n(A) &= 5 \\ n(B) &= 5 \\ n(A \cap B) &= 3 \\ n(A \cup B) &= 7 \\ n(A') &= 4 \\ n(A \cap B') &= 2 \\ n(\varnothing) &= 0 \end{aligned}$$

Example

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Let the universal set U be the set of positive integers less than or equal to 100. Let A be the set of multiples of 3 in U, and let B be the set of multiples of 5 in U.

(a) *Find $n(A \cap B)$, $n(A \cap B')$, $n(B \cap A')$, and $n(A' \cap B')$.*

(b) *Draw a Venn diagram with circles labeled A and B, indicating the numbers of elements in the subsets of part (a).*

Now You Try It!

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Let the universal set U be the set of positive integers less than or equal to 100. Let A be the set of multiples of 4 in U, and let B be the set of multiples of 7 in U.

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Solution

(a) $n(A \cap B) = 3$, $n(A \cap B') = 22$, $n(B \cap A') = 11$, and $n(A' \cap B') = 64$.

Basic Counting

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Addition Principle

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Now, suppose that every freshmen who is majoring in Chemistry is enrolled in Calculus or in History. If there are 20 freshmen Chemistry majors enrolled in Calculus and 15 freshmen Chemistry majors enrolled in History. How many total freshmen Chemistry majors are there?

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Theorem (Addition Principle for Counting)

For any two sets A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Now You Try It!

Example

According to a survey of business firms in a certain city, 345 *firms offer their employees group life insurance,* 285 *offer long-term disability insurance, and* 115 *offer group life insurance and long-term disability insurance. How many firms offer their employees group life insurance or long-term disability insurance?*

Now You Try It!

Example

According to a survey of business firms in a certain city, 345 firms offer their employees group life insurance, 285 offer long-term disability insurance, and 115 offer group life insurance and long-term disability insurance. How many firms offer their employees group life insurance or long-term disability insurance?

Solution

515

Multiplication Principle

Example

Suppose a store has 3 *types of shirts, and in each type of shirt, they have* 4 *colors available. How many options are available?*

Multiplication Principle

Theorem (Multiplication Principle for Counting)

1. *If two operations O_1 and O_2 are performed in order, with N_1 possible outcomes for the first operation and N_2 possible outcomes for the second operation, then there are*

$$N_1 \cdot N_2$$

possible combined outcomes of the first operation followed by the second operation.

2. *In general, if n operations $O_1, O_2, ..., O_n$ are performed in order, with possible number of number of outcomes $N_1, N_2, ..., N_n$, respectively, then there are*

$$N_1 \cdot N_2 \cdots N_n$$

possible combined outcomes of the operations performed in the given order.

Now You Try It!

Example

Suppose a 6-sided die and a 12-sided die are rolled. How many different possible outcomes are there?

Now You Try It!

Example

Suppose a 6-sided die and a 12-sided die are rolled. How many different possible outcomes are there?

Solution

72

More Multiplication Principle

Example

Suppose we have a list of 8 *letters that we wish to make code words from. How many possible* 4*-letter code words can be made if:*

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Now You Try It!

Example

Suppose we have a list of 10 *letters that we wish to make code words from. How many possible* 5*-letter code words can be made if:*

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Now You Try It!

Example

Suppose we have a list of 10 *letters that we wish to make code words from. How many possible* 5*-letter code words can be made if:*

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Solution

(a) $100,000$*, (b)* $30,240$*, (c)* $65,610$

Combining Rules

Example

There are 30 *teams in the MLB. Suppose a store sells both fitted and snapback baseball caps. Suppose the store carries standard and alternate versions of the fitted cap for each team, but only the standard version of the cap for the snapback cap. How many total different baseball caps do they sell?*